Det Kgl. Danske Videnskabernes Selskab.
Mathematisk-fysiske Meddelelser. **VII,** 2.

ON BEAT-PHENOMENA IN CYLINDRICAL TUBES EXPOSED TO SOUND-WAVES

BY

JUL. HARTMANN AND BIRGIT TROLLE

WITH THREE PLATES

KØBENHAVN
HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI
1925

Pris: Kr. 2,85.

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Introduction.

In measuring the length of short sound-waves by means of Kundt-tubes it appeared that if tubes of rather great width were used, i. e. tubes with diameters greater than abt. $^2/_3$ of the wave-length, dust-figures, entirely different from the ordinary Kundt-figures, were produced. Either these latter figures quite disappeared, being replaced by dust-heaps at greater distance from each other, or, in intermediate cases, the Kundt-figures appeared periodically more or less distinctly developed, the dust forming a band of periodically varying width, furrowed crosswise by the intervals between the Kundt heaps.

A preliminary examination of these new dust-figures, the K-figures, as we have chosen to call them, showed:

1.—The K-figures are preferably formed at the mouth of the tube, contrary to the Kundt-figures which appear, in the case of a closed tube, most distinctly at the farther end.
2.—With constant wave-length the distance between the K-figures increases as the square of the diameter of the tube.
3.—With constant diameter of the tube the distance varies inversely as the wave-length or nearly so.

These results combined with theoretical considerations led us to the conclusion that the phenomenon must be due to interference between waves proceeding in the same direction and not as with the ordinary Kundt-waves between waves in opposite directions. Thus in our opinion the origin of the K-figures should be a beat-phenomenon. The following investigation was carried out in order to settle the question as to the nature of the new figures.

The theory of wave-formation in cylindrical tubes.

The hydrodynamical equations may, with cylindrical coordinates be written

$$1^0 \qquad \varrho\left(\frac{\partial \varphi_r}{\partial r}+\frac{\varphi_r}{r}+\frac{\partial \varphi_n}{r\partial \theta}+\frac{\partial w}{\partial z}\right)+\frac{d\varrho}{dt}=0$$

$$2^0 \qquad \frac{\partial P}{\partial r}+\varrho\frac{d\varphi_r}{dt}=0$$

$$3^0 \qquad \frac{\partial P}{r\partial \theta}+\varrho\frac{d\varphi_n}{dt}=0$$

$$4^0 \qquad \frac{\partial P}{\partial z}+\varrho\frac{dw}{dt}=0$$

$$5^0 \qquad \frac{P}{P_0}=\left(\frac{\varrho}{\varrho_0}\right)^k,$$

where r, θ and z designate the coordinates, ϱ the density and P the pressure of the air, while φ_r, φ_n and w indicate the components of the velocity in the direction of the radius, a line perpendicular to the latter and in the direction of the axis of the tube. It should be borne in mind that

$$6^0 \qquad \frac{d}{dt}=\frac{\partial}{\partial t}+\varphi_r\frac{\partial}{\partial r}+\varphi_n\frac{\partial}{r\partial \theta}+w\frac{\partial}{\partial z}.$$

In considering vibratory movements of the air with amplitudes so small as to allow the neglect of terms of second order in the variations, the equations may be solved in putting:

$$7^0 \quad \varrho = \varrho_o (1 + n e^I), \qquad 8^0 \quad P = P_o (1 + p e^I), \qquad 9^0 \quad \varphi_r = \overline{\varphi}_r e^I,$$
$$10^0 \quad \varphi_n = \overline{\varphi}_n e^I, \qquad 11^0 \quad w = \overline{w} e^I$$

where
$$12^0 \quad I = i(vt + \mu\theta + \gamma z)$$

n, p, $\overline{\varphi}_r$, $\overline{\varphi}_n$ and $\overline{w}$ being functions of r, while ϱ_o and P_o denote the normal values of ϱ and P, and v indicates the cyclic frequency of the vibrations.

Introducing these expressions and neglecting terms of higher order one gets:

$$13^0 \qquad \dot{\overline{\varphi}}_r + \frac{\overline{\varphi}_r}{r} + \frac{i\mu}{r} \cdot \overline{\varphi}_n + i\gamma\overline{w} + ivn = 0$$

$$14^0 \qquad \overline{\varphi}_r = i\frac{b}{v} \cdot \dot{p}$$

$$15^0 \qquad \overline{\varphi}_n = -\frac{\mu b}{rv} \cdot p$$

$$16^0 \qquad \overline{w} = -\frac{\gamma b}{v} \cdot p$$

$$17^0 \qquad n = \frac{1}{k} \cdot p$$

where $b = \frac{P_o}{\varrho_o} = \frac{c^2}{k}$, c being the velocity of sound.

If a velocity-potential

$$18^0 \qquad \Phi = \overline{\Phi} \cdot e^I = \overline{\Phi} e^{i(vt + \mu\theta + \gamma z)}$$

exists we have:

$$19^0 \quad \overline{\varphi}_r = -\frac{\partial \Phi}{\partial r}, \qquad 20^0 \quad \overline{\varphi}_n = -\frac{\partial \Phi}{r \partial \theta}, \qquad 21^0 \quad \overline{w} = -\frac{\partial \Phi}{\partial z}$$

Finally we get from 13⁰—17⁰

22⁰ $$p = -\frac{iv}{b} \cdot \Phi.$$ and

23⁰ $$\ddot{\overline{\Phi}} + \frac{1}{r}\dot{\overline{\Phi}} + \left(\frac{v^2}{kb} - \gamma^2 - \frac{\mu^2}{r^2}\right)\overline{\Phi} = 0.$$

From the latter equation follows that $\overline{\Phi}$ is the general Besselfunction;

24⁰ $$\overline{\Phi} = AJ_{\mu}(hr) + BY_{\mu}(hr)$$

where

25⁰ $$h^2 = \frac{v^2}{c^2} - \gamma^2 = \frac{4\pi^2}{\lambda^2} - \gamma^2.$$

λ being the wave-length of a plane wave of frequency v.

Now, in order to determine which harmonic waves of frequency v there may actually appear in a cylindrical tube the boundary conditions must be taken into account. These conditions are:

1.—P should be finite for $r = 0$, from which follows that $B = 0$.

2.—P should have the same value for θ and for $\theta + 2p\pi$. thus μ must be an integer.

3.—$\overline{\varphi}_r = -\frac{\partial \overline{\Phi}}{\partial r} = -Ah\dot{J}_{\mu}(hr)$ should be 0 at the wall of the tube, i. e. for $r = \frac{d}{2}$, from which follows that $h \cdot \frac{d}{2}$ must be one of the roots: $q_{\mu,1}\, q_{\mu,2}\, q_{\mu,3} \cdots q_{\mu,n}$ in $\dot{J}_{\mu}(x) = 0$.

Thus h, and therewith γ is determined by:

26⁰ $$h^2_{\mu,n} = \frac{4q^2_{\mu,n}}{d^2} = \frac{4\pi^2}{\lambda^2} - \gamma^2_{\mu,n}; \quad \gamma^2_{\mu,n} = 4\left(\frac{\pi^2}{\lambda^2} - \frac{q^2_{\mu,n}}{d^2}\right).$$

Any wave of the type

27⁰ $$J_{\mu}\left(q_{\mu,n} \cdot \frac{2r}{d}\right) \cdot e^{i(vt + \mu\theta \pm \gamma_{\mu,n} \cdot z)}$$

where $\gamma^2_{\mu,n} = 4\left(\frac{\pi^2}{\lambda_2} - \frac{q^2_{\mu,n}}{d^2}\right)$ and μ an integer may thus appear in the tube.

It being furthermore known that any possible vibratory conditions at or rather in the mouth of the tube may be represented by a series of J-functions, like those above, it is obvious that a tube exposed to harmonic sound-waves of a frequency v, will generally be passed by the waves represented by:

$$28^0 \qquad \Phi = e^{ivt} \sum^{\mu} \sum^{n} A_{\mu,n} J_{\mu}\left(q_{\mu,n} \cdot \frac{2r}{d}\right) \cdot e^{i(\mu\theta \pm \gamma_{\mu,n} z)}.$$

The component waves.

We now proceed to consider the component terms of the series above. The general term is:

$$\Phi = A \cdot J_{\mu}\left(q_{\mu,n} \frac{2r}{d}\right) e^{i(vt + \mu\theta \pm \gamma_{\mu,n} z)}.$$

$q_{\mu,n}$ being a root in $\dot{J}_{\mu}(x) = 0$, μ an integer and

$$\gamma_{\mu,n} = 2\sqrt{\frac{\pi^2}{\lambda^2} - \frac{q^2_{\mu,n}}{d^2}}.$$

Obviously, if $\gamma_{\mu,n}$ is real i. e. if $\frac{d}{\lambda} > \frac{q_{\mu,n}}{\pi}$, the term represents a wave proceeding along the axis of the tube. The wave-length in the direction of the axis is determined by:

$$\gamma_{\mu,n} \cdot \lambda_{\mu,n} = 2\pi, \quad \text{i. e.} \quad \lambda_{\mu,n} = \frac{\lambda}{\sqrt{1 - \frac{\lambda^2 q^2_{\mu,n}}{\pi^2 d^2}}}$$

and the velocity in the same direction by:

$$C_{\mu,n} = \frac{c}{\sqrt{1 - \frac{\lambda^2 \cdot q^2_{\mu,n}}{\pi^2 d^2}}} > c.$$

If $\gamma_{\mu,n}$ is imaginary, i. e. $\frac{d}{\lambda} < \frac{q_{\mu,n}}{\pi}$ a singular type of motion appears which, however, will not be discussed in this paper.

The J_0-waves. The waves corresponding to $\mu = 0$ are represented by:

$$\Phi = A \cdot J_0\left(q_{0,n} \cdot \frac{2r}{d}\right) e^{i(vt \pm \gamma_{0,n} z)}$$

Being independent of θ, all the waves of this order are symmetrical with regard to the axis. The roots of $\dot{J}_0(x) = 0$ are 0, 3.8317, 7.0156 etc.

The wave corresponding to $q_{0,1}$ is:

$$\Phi = A e^{iv\left(t \pm \frac{z}{c}\right)}$$

representing an ordinary plane wave with an amplitude independent of r and θ and with a velocity c equal to that of a free plane wave.

To $q_{0,2} = 3.8317$ corresponds

$$\Phi = AJ_0\left(3.8317 \cdot \frac{2r}{d}\right) e^{i\left(vt \pm \frac{z}{c}\sqrt{1-\left(\frac{\lambda}{\pi} \cdot \frac{3.8317}{d}\right)^2}\right)}.$$

If $\frac{d}{\lambda} > \frac{3.8317}{\pi} = 1.2197$ this wave, runs down the axis with a wavelength:

$$\lambda_{0,2} = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda \cdot 3.8317}{\pi d}\right)^2}}.$$

The first root in $J_0(x) = 0$, $x = 2.4042$, being less than $q_{0,2}$, p and w alter their signs for $r = d \cdot \frac{2.4042}{2 \cdot 3.8317}$ and the cross-section is divided by this radius into 2 concentric parts for which p and w are in opposite phases. The velocity φ_r which varies as $\dot{J}_0\left(3.8317 \cdot \frac{2r}{d}\right)$ has the same direction and phase all over the cross-section. Fig. 1, Pl. I, illustrates the distribution of radial and axial velocity together with the deplacements, at a given moment, over part of the tube.

Considering finally the general term

$$\Phi = A \cdot J_0\left(q_{0,n} \cdot \frac{2r}{d}\right) \cdot e^{i(vt \pm \gamma_{0,n} z)}$$

it is obvious that, J_0 having $n-1$ roots between $r=0$ and $r = \frac{d}{2}$, the cross-section is divided into n concentric parts of which any neighbouring sections are in opposite vibratory conditions with regard to p and w. Thereby it is always assumed that $\frac{d}{\lambda} > \frac{q_{0,n}}{\pi}$. However, the greater $q_{0,n}$, the less is the chance that the said condition will be fullfilled and for a certain number n the wave becomes one of the singular waves indicated above.

The J_1-waves. The waves corresponding to $\mu = 1$ are represented by:

$$\Phi = A \cdot J_1\left(q_{1,n} \cdot \frac{2r}{d}\right) \sin(\theta - \theta_0)\, e^{i(vt \pm \gamma_{1,n} z)}.$$

The roots of $\dot{J}_1(x) = 0$ are: 1.8412, 5.3314 ... etc. The tube is now by the plane $\theta = \theta_0$ divided into two parts for which p, w and φ_r are in opposite phases, the latter vibrations having a plane of symmetry: $\theta = \theta_0 + \frac{\pi}{2}$. $J_{1,n}\left(q_{1,n} \cdot \frac{2r}{d}\right) = 0$ having $n-1$ roots $< q_{1,n}$ the cross-section is furthermore by $n-1$ circles divided into n parts of which any consecutives are of opposite phase as to p and w. Figs. 2 A & B, Pl. II & III, are drawn to convey an idea of the vibratory conditions at a certain moment for the $J_1\left(q_{1,1} \cdot \frac{2r}{d}\right)$-wave.

The J_μ-waves. It is now quite obvious that the cross-section with the wave:

$$\Phi = A \cdot J_{\mu}\left(q_{\mu,n} \cdot \frac{2\,r}{d}\right) e^{i\theta\mu} \cdot e^{i(vt \pm \gamma_{\mu,n} z)}$$

is divided by μ diametrical planes into 2μ parts. Passing from one part to the following the p-, w- and φ_r-vibrations alter their signs. Furthermore the cross-section is divided into n zones by the $n-1$ circles corresponding to the first $n-1$ roots of

$$J_{\mu}\left(q_{\mu,n} \frac{2\,r}{d}\right) = 0,$$

neighbouring zones having opposite phases with regard to p and w.

The waves to be expected in a given tube.

Generally all waves, for which $\frac{d}{\lambda} > \frac{q_{\mu,n}}{\pi}$, may be expected. However, if the source of sound is equal in all directions and situated in the axis of the tube, only the J_0-waves can develop because of the symmetry. On the other hand, if the source of sound is outside the axis, all the waves may generally be anticipated. They will, in this case, have a plane of symmetry containing the source of sound and the axis.

In view of the determination of the possible waves the roots of $\dot{J}_{\mu}(x)$ are arranged according size in the following table:

Tab. I.

	$q_{\mu,n}$	$\frac{q_{\mu,n}}{\pi}$
$q_{0,1}$	0	0
$q_{1,1}$	1.8412	0.5861
$q_{2,1}$	3.0542	0.9722
$q_{0,2}$	3.8317	1.2197
$q_{3,1}$	4.2021	1.3376
$q_{4,1}$	5.3176	1.6926
$q_{1,2}$	5.3314	1.6970
$q_{5,1}$	6.4155	2.0421
$q_{2,2}$	6.7061	2.1346
$q_{0,3}$	7.0156	2.2330

Thus if $\frac{d}{\lambda} < 0.5861$ only the plane wave may be expected giving the ordinary Kundt-figures. If $0.586 < \frac{d}{\lambda} < 0.972$ the wave $J_1\left(q_{1,1}\frac{2r}{d}\right)$ may appear in addition to the plane wave, and if the two waves are of approximately equal intensity in the plane of symmetry of the J_1-wave vigorous beats must develop in the said plane and cause the dust to gather where the two waves compensate each other. In the case of $0.972 < \frac{d}{\lambda} < 1.219$ there is a possibility of getting the wave $J_2\left(q_{2,1}\cdot\frac{2r}{d}\right)$ in addition to the two waves already mentioned. Beat-phenomena of fairly great complexity must then be anticipated, the phenomena depending on the ratio of intensity of the waves and of their phase-differences. If finally $1.219 < \frac{d}{\lambda} < 1.337$ the wave $J_0\left(q_{0,2}\cdot\frac{2r}{d}\right)$ is furthermore added and the beat-phenomenon will generally be very complex. However if the source of sound is adjusted in the axis and is symmetrical relative to the latter, only the plane wave and $J_0\left(q_{0,2}\cdot\frac{2r}{d}\right)$ can develop and there is a chance of obtaining simpler beat-phenomena.

Beats between two waves.

When two of the waves considered above are running down the tube, the waves having the same frequency but different wave-lengths, beats must occur. The distance between two points of reinforcement is determined by holding one more wave-length of the one wave than of the other. Thus:

$$K^{\mu_1, n_1}_{\mu_2, n_2} = m \cdot \lambda_{\mu_1, n_1} = (m-1)\lambda_{\mu_2, n_2} = \frac{1}{\dfrac{1}{\lambda_{\mu_1, n_1}} - \dfrac{1}{\lambda_{\mu_2, n_2}}}$$

$$= \frac{\lambda}{\sqrt{1-\left(\dfrac{\lambda q_{\mu_1, n_1}}{\pi d}\right)^2} - \sqrt{1-\left(\dfrac{\lambda q_{\mu_2, n_2}}{\pi d}\right)^2}}$$

$$= \frac{d^2}{\lambda\left[\left(\dfrac{q_{\mu_2, n_2}}{\pi}\right)^2 - \left(\dfrac{q_{\mu_1, n_1}}{\pi}\right)^2\right]} \cdot \left[\sqrt{1-\left(\dfrac{\lambda q_{\mu_1, n_1}}{\pi d}\right)^2} + \sqrt{1-\left(\dfrac{\lambda q_{\mu_2, n_2}}{\pi d}\right)^2}\right].$$

If the two wave-lengths are nearly equal this may be written:

$$K^{\mu_1, n_1}_{\mu_2, n_2} = \frac{2}{\left(\dfrac{q_{\mu_2, n_2}}{\pi}\right)^2 - \left(\dfrac{q_{\mu_1, n_2}}{\pi}\right)^2} \cdot \frac{d^2}{\lambda} = C \cdot \frac{d^2}{\lambda}$$

thus representing just the dependency found in the preliminary experiments. Some values for C in the formula:

$$K = C\frac{d^2}{\lambda}$$

and calculated from:

$$C = \frac{2}{\left(\dfrac{q_{\mu_2, n_2}}{\pi}\right)^2 - \left(\dfrac{q_{\mu_1, n_1}}{\pi}\right)^2}$$

are quoted in the following table:

$C^{0,1}_{1,1} = 5.824$	$C^{0,1}_{0,2} = 1.344$
$C^{0,1}_{2,1} = 2.116$	$C^{1,1}_{0,2} = 1.748$
$C^{1,1}_{2,1} = 3.324$	$C^{0,1}_{3,1} = 1.118$

However, as shown by the correct formula, C is not a real constant, it varies somewhat with the ratio $\frac{d}{\lambda}$.

Values of $C^{0,1}_{1,1}$ corresponding to beats between the plane wave and the $J_1\left(q_{1,1}\frac{2r}{d}\right)$-wave are tabulated in the following:

$\frac{d}{\lambda}$	$C^{0,1}_{1,1}$	$\frac{d}{\lambda}$	$C^{0,1}_{1,1}$
∞	5.824	0.833	4.983
2.00	.696	.769	.796
1.67	.636	.714	.577
1.43	.568	.667	.301
1.25	.484	.625	3.960
1.11	.386	.588	.469
1.00	.272	.586	2.912
.909	.139		

The beats corresponding to the smaller values of C are generally of inconsiderable intensity.

Final experiments.

The beat-phenomenon was originally observed in working with the air-jet generator invented by one of the authors.[1] With the same generator, by which waves of great intensity can be produced, all the final observations were made. In order to secure constant frequency of the wave, the generator must be worked with air of constant pressure. In the experiments here considered, the air was furnished from a steel bottle of abt. 20 l containing air of up to a pressure of abt. 100 atm. The pressure was reduced in two stages by means of reduction-valves, each valve opening into an air-chamber. With this arrangement the frequency could be kept almost constant for several hours.

The generator G was vertically arranged as indicated in fig. 3. The tube T was adjusted horizontally. It was fastened to a slide by means of which it could be moved in a vertical plane containing the source of sound. By this arrangement

[1] Det kgl. Danske Videnskabernes Selskabs Meddelelser I, 13. 1919 and The Phys. Rev. Vol. XX. 719. 1922.

the dust-band came in the plane of symmetry i. e. in that part of the tube where the intensest and simplest beat phenomena were to be expected. The tubes were now rather long, abt. $^2/_3$ m., both ends were open. They were of such a length that even if the farther end were closed and the tubes exposed to waves of such a length that the ordinary Kundt-figures could be anticipated these latter figures did not appear. Nevertheless the number of K-figures was increased very considerably compared with the figures in the formerly used short and closed tubes. It appeared, however, that it was of small or no consequence whether or not the $^2/_3$ m. tubes were closed.

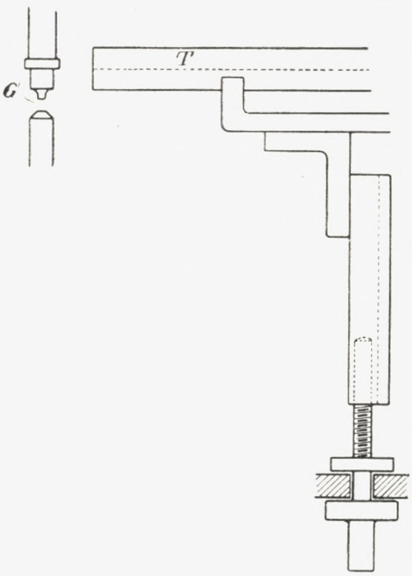


Fig. 3.

These observations made it probable that the K-figures originate in a beat-phenomenon and not in interference between waves running opposite.

The experiments were carried out with various tubes, the level of the tube being varied gradually relative to the generator as already indicated.

In order to illustrate the experiments, the observations made with a tube of diameter 0.5 cm. exposed to waves of length 0.59 cm. will now be described. With this tube $\frac{d}{\lambda} = 0.84$. From tab. 1 it is seen that in this case only the plane wave and the wave $J_1\left(q_{1,1}\frac{2\,r}{d}\right)$ may be expected. The K-figures were observed at various levels of the tube and at every level with various distances of the mouth from generator. The latter distance proved, however, of small influence on the position of the K-figures relative to the tube. Also the level of the tube proved rather unimportant pro-

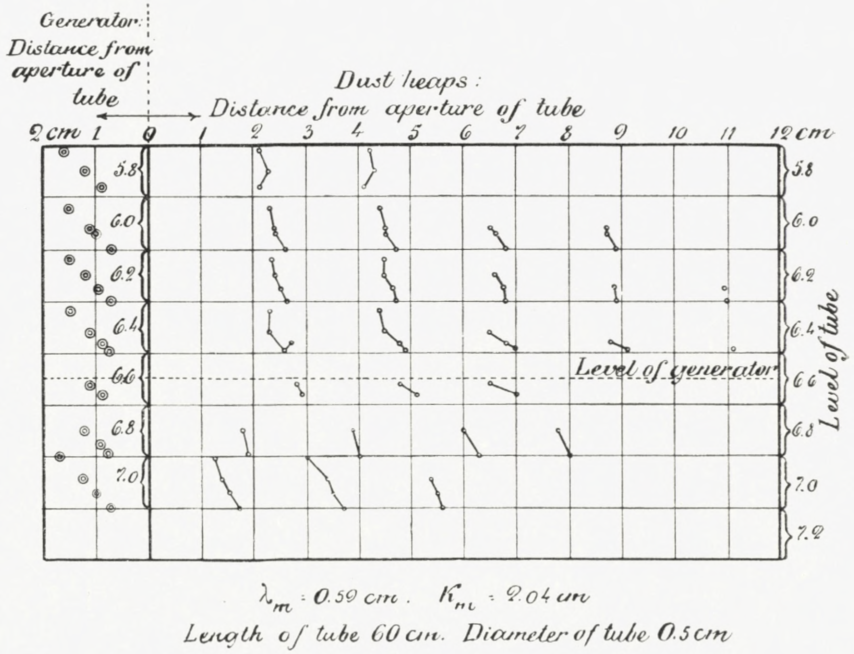


Fig. 4.

vided that the tube, when moved downwards, had not yet passed the level of the generator. With the tube at this level only few K-figures were observed and the figures only appeared with the tube at a short distance from the generator. When the level of the generator had been passed the figures reappeared and were now to be found nearly midway between the former positions exactly as was to be expected from the theory above. Fig. 4 shows the position of the K-figures relative to the aperture of the tube — situated in the vertical line *0* —. The generator was at a level of 6.6. At every level of the tube observations were taken for three or four distances from the generator the position of the latter being indicated in fig. 4 by circles, and the positions of the dust-heaps by dots. It should be noticed that the distribution of the wave-energy round the generator was not quite symmetrical, the energy in the upward direction being somewhat in excess.

The conditions with a tube of diameter 0.625 cm., exposed to a wave of length 0.575 cm. making $\frac{d}{\lambda}$ equal to 1.08, were quite similar to those of the former tube, although there was in this case a slight possibility of getting the wave $J_2\left(q_{2,1}\cdot\frac{2r}{d}\right)$ too.

With a tube of 0.72 cm. exposed to waves of 0.59 cm., $\frac{d}{\lambda}$ thus being 1.22, the main aspect of the figures was the same as before. Only with the tube in extreme positions relative to the level of the generator, some dust-heaps were added, undoubtedly originating from the wave $J_2\left(q_{2,1}\cdot\frac{2r}{d}\right)$.

With a tube of diameter 0.85 cm. and a wave-length 0.57 cm., thus $\frac{d}{\lambda} = 1.49$, the waves $J_0\left(q_{0,2}\cdot\frac{2r}{d}\right)$ and $J_3\left(q_{3,1}\cdot\frac{2r}{d}\right)$ might be anticipated in addition to the three waves already mentioned. In accordance herewith rather intricate dust-figures were observed. However, with the tube on a level with the generator a simple set of dust-heaps were observed corresponding to beats between the plane wave and the wave $J_0\left(q_{0,2}\cdot\frac{2r}{d}\right)$. The latter alone can develop in the case of the boundary-conditions being symmetrical relative to the axis. Beneath and above the level of the generator the plane wave and the wave $J_1\left(q_{1,1}\cdot\frac{2r}{d}\right)$ were of the highest intensity and the corresponding K-figures could always be distinguished, but in addition to the said figures several others appeared.

With a tube of 1.15 cm. exposed to waves of 0.57 cm. wave-length, making $\frac{d}{\lambda} = 2.0$, the two waves $J_4\left(q_{4,1}\cdot\frac{2r}{d}\right)$ and $J_1\left(q_{1,2}\cdot\frac{2r}{d}\right)$ were added to those already mentioned. Accordingly the picture was in general perplexingly complicated. However, here, as in the foregoing case with the tube on a level with the generator, the figure became simple, consisting of 19 dust-heaps equidistantly arranged and corresponding to beats between the plane wave and the wave $J_0\left(q_{0,2}\cdot\frac{2r}{d}\right)$ which is still the only wave beside the plane wave symmetrical relative to the axis.

In addition to the experiments here mentioned several others were carried out with the same or other tubes and with varied wave-lengths, most of them with the aim of determining C in the formula:

$$K = C\cdot\frac{d^2}{\lambda}.$$

From beats between the plane wave and $J_1\left(q_{1,1}\cdot\frac{2r}{d}\right)$ the following results were obtained:

$\frac{d}{\lambda}$	C_{obs}	C_{cal}
0.82	4.82	4.94
0.84	4.94	5.00
0.85	4.81	5.01
1.12	5.12	5.40
1.14	4.92	5.41
1.22	5.06	5.46
1.50	5.13	5.59
2.01	5.73	5.70

From beats between the plane wave and $J_0\left(q_{0,2}\cdot\frac{2r}{d}\right)$ the following results were derived:

$\frac{d}{\lambda}$	C_{obs}	C_{cal}
2.01	1.20	1.17
1.50	1.00	1.06

In spite of the waves emitted from the generator not being quite harmonic, the wave with double frequency being rather pronounced, no effect of the over-tones was observed.

The Royal Technical Highschool
Physical Laboratory II
Copenhagen.

We owe thanks to the board of the Carlsbergfund who made the above investigation possible by a subvention.

Færdig fra Trykkeriet d. 29. Maj 1925.

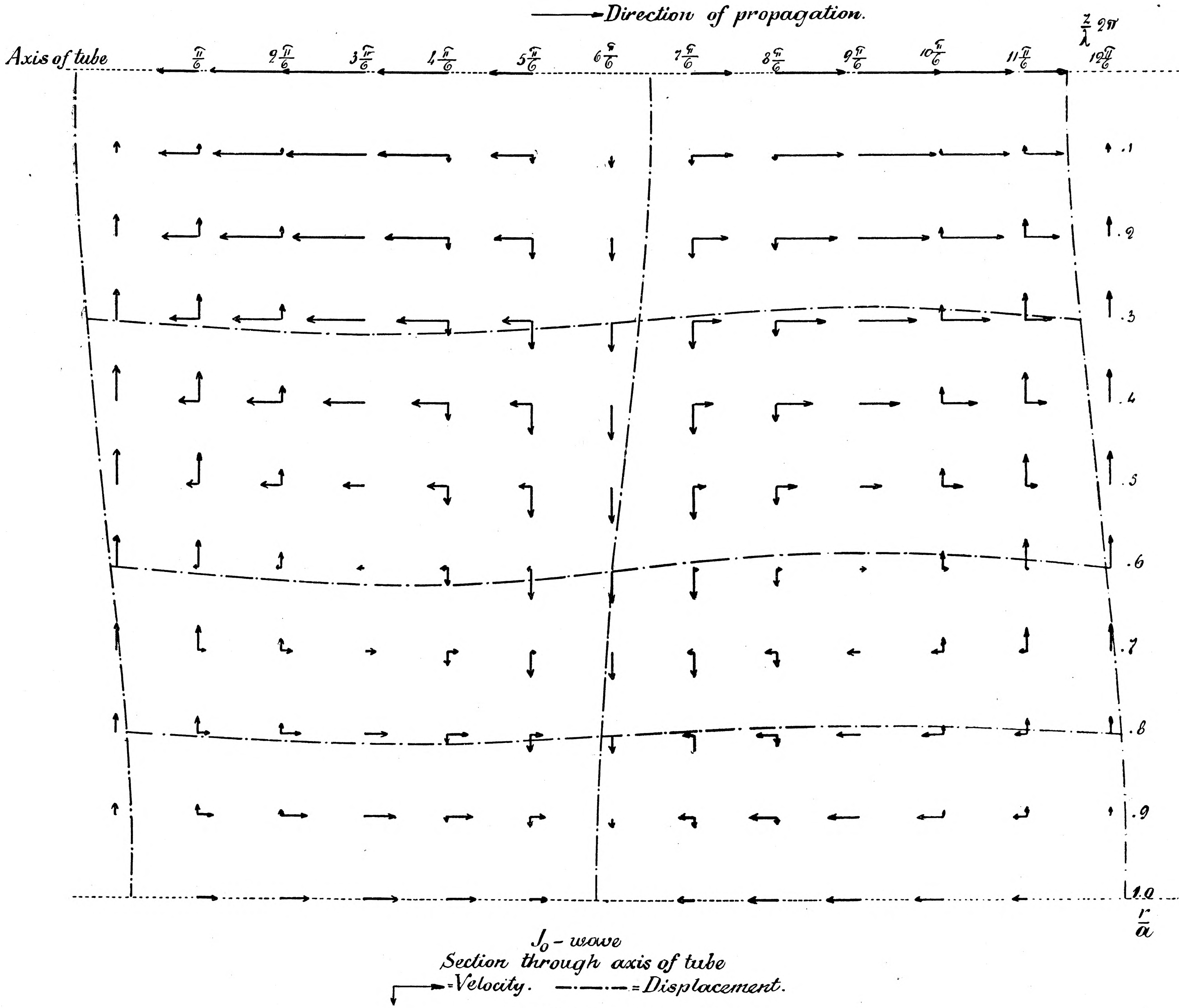
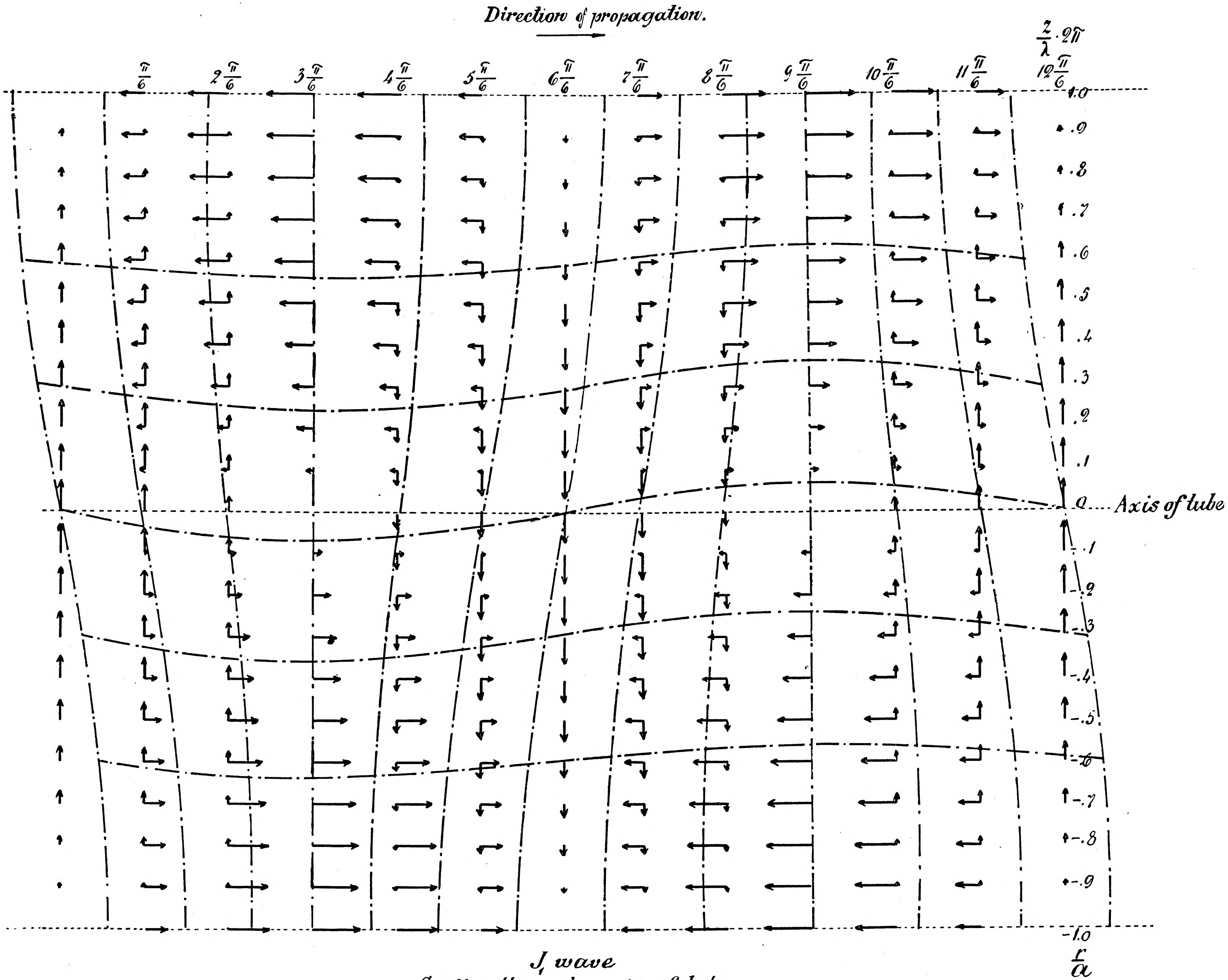
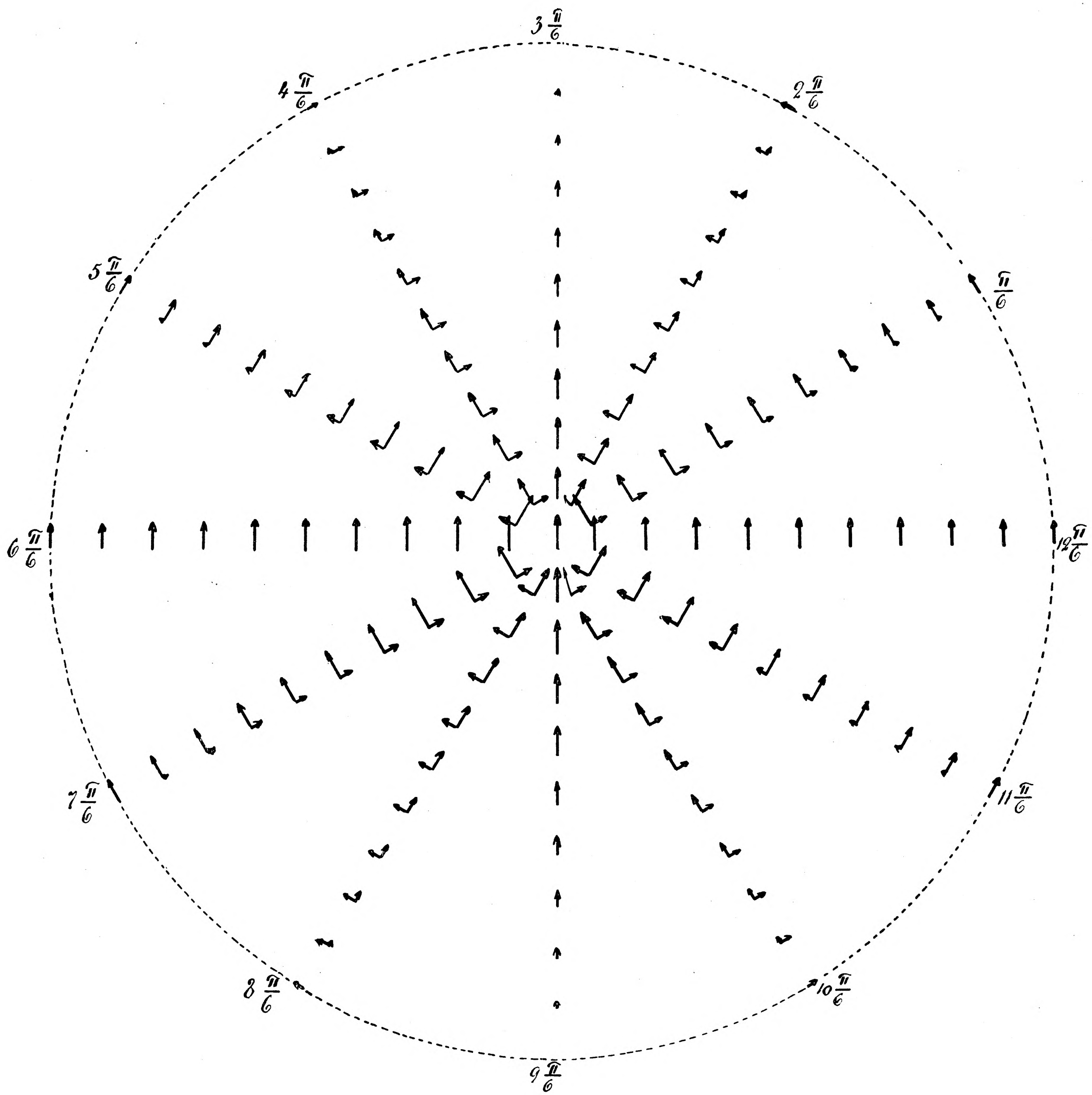

Direction of propagation.
Axis of tube
$\frac{z}{\lambda} 2\pi$
$\frac{\pi}{6}$ $2\frac{\pi}{6}$ $3\frac{\pi}{6}$ $4\frac{\pi}{6}$ $5\frac{\pi}{6}$ $6\frac{\pi}{6}$ $7\frac{\pi}{6}$ $8\frac{\pi}{6}$ $9\frac{\pi}{6}$ $10\frac{\pi}{6}$ $11\frac{\pi}{6}$ $12\frac{\pi}{6}$
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\frac{r}{a}$
J_0 - wave
Section through axis of tube
= Velocity. = Displacement.

Fig. 1, $n = 2$.



J₁ wave
Section through axis of tube
= *Velocity*. —·—·— = *Displacement*.

Fig. 2 *A*, $n = 1$.



J_1-wave
Section perpendicular to axis.

Fig. 2*B*, $n = 1$.

MATHEMATISK-FYSISKE MEDDELELSER

UDGIVNE AF

DET KGL. DANSKE VIDENSKABERNES SELSKAB

4. BIND (Kr. 13,20):

		Kr. Ø.
1.	Nielsen, Niels: Recherches sur l'Équation de Fermat. 1922	5.75
2.	Jacobsen, C. & Olsen, Johs.: On the Stopping Power of Lithium for α-Rays. 1922	0.60
3.	Nørlund, N. E.: Nogle Bemærkninger angaaende Interpolation med æquidistante Argumenter. 1922	1.10
4.	Brønsted, J. N.: The Principle of the Specific Interaction of Ions. 1921	1.15
5.	Pedersen, P. O.: En Metode til Bestemmelse af den effektive Modstand i højfrekvente Svingningskredse. 1922	0.70
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7.	Pedersen, P. O.: On the Lichtenberg Figures. Part II. 1. The distribution of the velocity in positive and negative figures. 2. The use of Lichtenberg figures for the measurement of very short intervals of time. With two plates. 1922	2.15
8.	Bøggild, O. B.: Re-Examination of some Zeolites (Okenite, Ptilolite, etc.). 1922	1.40
9.	Wiedemann, E. und Frank, J.: Über die Konstruktion der Schattenlinien auf horizontalen Sonnenuhren von Tâbit ben Qurra. 1922	0.75
10.	Pedersen, P. O.: Om elektriske Gnister. I. Gnistforsinkelse. Med 2 Tavler. 1922	3.25

5. BIND (Kr. 13,10):

		Kr. Ø.
1.	Nielsen, Niels: Recherches sur les Équations de Lagrange. 1923	3.20
2.	Kampé de Fériet, J.: Sur une formule d'addition des Polynomes d'Hermite. 1923	0.50
3.	Hansen, H. M., Takamine, T., and Werner, Sven: On the Effect of Magnetic and Electric Fields on the Mercury Spectrum. With two plates and figures in the text. 1923	2.25
4.	Nielsen, Niels: Recherches sur certaines Équations de Lagrange de formes spéciales. 1923	3.00
5.	Nielsen, Niels: Sur le genre de certaines Équations de Lagrange. 1923	2.25

6. BIND (KR. 17,00):

7. BIND: